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The Continuous Network Design Problem under the Traffic Network with Guidance System

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Abstract

Route guidance and information systems are developed and popularly applied to help travelers to make route decisions today. Such devices can present information or give recommendations, so they have influence on the travelers travel choice in the traffic network. In this sense, the influence of guidance system on travelers' route choice should be considered when the scenario of traffic network design is developed. However, few of existing traffic network design problem model considers the effect, and this motivates us to develop a Continuous Network Design (CNDP) model under a traffic network with guidance system. This study is based on the research of Jahn et al. (2005) to develop a single level program for CNDP under the traffic network with guidance system. The guidance system give advices to travelers based on system optimal rules with user constrains. From the result of simulation, it can be inferred that if the tolerance factor φ is properly chosen in our CNDP model, travelers all can decrease their travel time after network design than that they experienced at user equilibrium before network enhancement. That is to say, every traveler can profit from our CNDP scenario for proper tolerance factor φ.

Keywords: continuous network design problem; guidance system; system optimal

1. Introduction

To satisfy the growing traffic demand in a transportation network, one way for the traffic authority

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usually adopts is to enhance the capacities of the existing congested links. In this case, how much additional capacity is to be added to each of these existing links is an interesting problem, trying to minimize the total system costs under limited expenditure, meanwhile taking into account for the route choice behavior of network user. This problem is referred to as the Continuous Network Design Problem (CNDP) that determines the optimal capacity enhancement (expressed by continuous decision variable) for a subset of existing links. There have been many studies about CNDP. Abdulaal and LeBlance (1979) proposed a bilevel programming model for CNDP whose lower level was determinate user equilibrium (DUE). Marcotte (1986) and Friesz et al. (1990) gave a general bilevel programming mode for CNDP where the lower level in Abdulaal and LeBlance (1979) was substituted by the variatioal inequality formulation of the DUE. Yang and Bell (1998a) presented a comprehensive paper for the network design problem including various models and algorithms. Under travelers' route choice behaviors based on logit-based Stochastic User Equilibrium (SUE), Davis (1994) gave an equivalent single level continuously differentiable mathematical program model for the CNDP with the SUE condition. Patriksson and Rockafellar (2002) converted the bilevel problem into a single level program using variational inequality for DUE. Their network design formulation is called a mathematical program with equilibrium constraints (MPEC). However, few of existing CNDP models studies considers the effect of guidance system on the route choice behaviors of travelers. Today, route guidance and information systems are developed and popularly applied to help travelers to make route decisions. Such devices can present information or give recommendations, so they have influence on the travelers or users' travel choice in the traffic network. In this sense, the influence of guidance system on travelers' route choice should be considered when the scenario of traffic network design is developed. This motivates us to develop a Continuous CNDP model considering the effect of guidance system on travelers.

About the influence of guidance system on the travelers' route choice, there exists many theoretic explorations. Emmerink (1995) and Emmerink (1998) made a research on the effect of Advanced Traveler System (ATIS) in the behavior of the users of a traffic system based on an equilibrium model that considers a stochastic network of two arcs. Later, Verhoef et al. (1996) using the same test traffic network analyzed the effect of the joint implementation of ATIS and road pricing. Zhang and Verhoef (2006) presented a model to study the influences of information in a monopolistic market of ATIS. Jahn et al. (2005) proposed a route guidance and information systems based on a system optimal model under user constraints to guide the route of travelers' choice. Their model overcame the inherent problem of system-optimal guidance, which is well-known that in a system-optimal pattern users may be assigned to considerably longer routes for the benefit of other. So the system-optimal flow is unfair for some users in the sense that users between the same Origin Destination pairs (OD) have different travel time. The essence of their study is that system-optimal assignment of traffic flow under user constraints leads to a better system performance than the user equilibrium, meanwhile guaranteeing superior fairness compared to the pure system optimum. Considering the effect of route guidance on traveler, it is based on the study of Jahn et al. (2005) to describe the route choice of travelers under route guidance. The lower level problem of bilevel CNDP model is described by a system-optimal under user constraints and the bilevel CNDP model can be converted into a single program. Using the model, we will study how to develop the user constraints to make user of the traffic system with route guidance benefit from the network design, meanwhile decrease the total travel cost at best.

The CNDP model under the traffic network with guidance system is given in Section 2. And a sample network is given in Section 3 to illustrate our model; Section 4 presents the conclusions and future study.

2. Formulation of CNDP of the traffic network with guidance system

We assume that all user of the traffic network use the guidance system and they actually accept the

recommended routes leading to a system optimal under user constraints. Although the assumption is very strong, Jahn et al. (2005) verified that the proper user constraints can optimize the total travel cost without compromising user acceptance. To describe our CNDP model, the notions used in this study are given following.

In this paper, $G=(N,A)$ denote a direct connected traffic network of a node set: N, a link set: A. W is the set of origin-destination (OD) pairs, R_w denote the set of routes between OD pair w, and R denote all routes in this network, $R=\bigcup_{w\in W} R_w$. v_a and c_a denote respectively the traffic flow and the capacity on link a, $t_a(\bullet)$ is the travel cost of link a. f_r^w, c_r^w are the path flow and path travel cost of path $r \in R_w, w \in W$. Namely,

$$c_r^w = \sum_{a\in A} t_a(v_a)\delta_{ar}^w, \quad r \in R_w, w \in W$$

where $\delta_{ar}^w = 1$ if link a is used by path r, and $\delta_{ar}^w = 0$ otherwise. q_w denote the traffic demand between OD pair w. y_a is the capacity enhancements in CNDP. $G_a(y_a)$ is the expanding cost function of link $a, \forall a \in A$ and B is the budget.

2.1. The bi-level CNDP model

To illustrate the characteristic of our CNDP model, the bilevel CNDP model is given at first; it is called General CNDP to differentiate from our CNDP model in this paper and is shown as follows:

$$\min_{\mathrm{y}} z(\mathrm{x,y}) = \sum_{a\in A} t_a(v_a(\mathrm{y}), y_a)v_a(\mathrm{y}) + \alpha \sum_{a\in A} G_a(y_a) \tag{1}$$

$$\sum_r f_r^w = q_w, \quad \forall r \in R_w, \quad w \in W \tag{2}$$

$$\underline{y_a} \le y_a \le \overline{y_a}, \quad \forall a \in A \tag{3}$$

where α is a scaling coefficient, which is used to covert units of expand cost into units of travel cost; and $v_a(\mathrm{y}), a \in A$, is the solver of DUE traffic assignment:

$$\min_{\mathrm{v}} \int_0^{v_a} t_a(\omega, y_a)d\omega \tag{4}$$

subject to

$$\sum_r f_r^w = q_w, \quad \forall r \in R_w, \quad w \in W \tag{5}$$

$$f_r^w \ge 0, \quad \forall r \in R_w, \quad w \in W \tag{6}$$

$$x_a = \sum_w \sum_k f_k^w \delta_{ar}^w, \quad \forall\, a \in A, k \in R_w, w \in W \tag{7}$$

It is to say that the route choice behavior of network users is user-optimal. This is different from the assumptions in this study, in which the users comply with the route guidance which leads to the system optimal under user constraints. The special model is given following.

2.2. CNDP model in traffic network with guidance system

As mentioned above, the system-optimal pattern is unfair as in a system-optimal pattern users may be assigned to considerably longer routes for the benefit of other. To make the route guidance based on system optimal accepted by users, it is necessary to control the unfairness. This unfairness can be measured by Loaded unfairness which is the ration of her experienced travel time to the experienced travel time of the fasted traveler for the same OD pair. The Loaded unfairness is not easy to control. Jahn et *al*. (2005) advised controlling this unfairness based on normal length instead of actual travel time. The normal length of a path is defined by a prior estimate of the real travel time. For example, it can be either

the free time of the path or its travel time in a user equilibrium. In detail, the travelers just choose these paths whose normal length is not much greater than that of a shortest path with respect to normal length. In other words, the travelers make their trips on a given set of path in system optimal rules. The given path set can be denoted by P_w^{φ} between OD pair w for a given tolerance factor φ, which can be used to measure the network users' tolerance of the quality of information recommended by the Guidance System. A path $r \in \mathrm{P}_w^{\varphi}$ is feasible if the normal length $\tau_r \leq \varphi T_w$, where τ_r is the normal length of path r and $T_w := \min_{r \in R_w} \tau_r$.

Under the user constraints based on normal length for the proper φ, the system-optimal route guidance can provide recommendations accepted by network users. Because the (1) and the users' route choice have the same objective function, so the bilevel CNDP model under traffic network with guidance system can be described by a single program. The model is described by equations (8)-(12).

$$\min_{\mathrm{y}} z(\mathrm{x},\mathrm{y}) = \sum_{a \in A} t_a(v_a(\mathrm{y}), y_a) v_a(\mathrm{y}) + \alpha \sum_{a \in A} G_a(y_a) \tag{8}$$

$$s.t. \quad \underline{y_a} \leq y_a \leq \overline{y_a}, \quad \forall a \in A \tag{9}$$

$$\sum_r f_r^w = q_w, \quad \forall r \in P_w, \quad w \in W \tag{10}$$

$$f_r^w \geq 0, \quad \forall r \in P_w, \quad w \in W \tag{11}$$

$$x_a = \sum_w \sum_k f_k^w \delta_{ar}^w, \quad \forall\, a \in A, k \in R_w, w \in W \tag{12}$$

2.3. Algorithm

To solve problem (8), we use a variant of the convex combination algorithm of Frank-Wolf. Because the normal length in this paper is the travel time in user equilibrium, meanwhile there are relations between it and link capacity enhancement. Therefore, a DUE assignment process is added to find feasible path set P_w^{φ} for each given φ. In following, the specific algorithm is shown.

Step 1: Initialization. Determine an initial set of link flow x^0 and capacity enhancement y^0.

Step 2: Direction Finding. Find d_x^n and d_y^n that solve the problems (13) and (14) respectively.

$$\begin{aligned} &\min \sum_{a \in A} \left(t_a(x_a^{n-1}) + x_a^{n-1} \frac{\partial t}{\partial x_a}\right) d_{x_a}^n \\ &s.t. \sum_{k \in \mathrm{P}^{rs}} f_k^{rs} = d^{rs}, \quad \forall r, s \in N \\ &\quad f_k^{rs} \geq 0, \qquad \forall r, s \in N \\ &\quad d_{x_a}^n = \sum_w \sum_k f_k^w \delta_{ar}^w, \quad \forall\, a \in A, k \in R_w, w \in W \end{aligned} \tag{13}$$

$$\begin{aligned} &\min \sum_a \left(x_a^{n-1} \frac{\partial t}{\partial y_a} + g(y_a^{n-1})\right) d_{y_a}^n \\ &s.t. \quad l_a \leq d_{y_a}^n \leq u_a, \quad \forall a \in A \end{aligned} \tag{14}$$

Step 3: Step-size determination. Find α^n that solves problem (15).

$$\begin{aligned} &\min \sum_a (x_a^{n-1} + \alpha^n (d_{x_a}^n - x_a^{n-1})) t_a (x_a^{n-1} + \alpha^n (d_{x_a}^n - x_a^{n-1})) \\ &\quad + \alpha^n \sum_a g_a (y_a^{n-1} + \alpha^n (d_{y_a}^n - y_a^{n-1})) \\ &s.t \quad 0 \leq \alpha^n \leq 1 \end{aligned} \tag{15}$$

Step 4: Move. $\mathrm{x}^{n-1} = \mathrm{x}^{n-1} + \alpha^n (\mathrm{d}_x^n - \mathrm{x}^{n-1})$, $\mathrm{y}^n = \mathrm{y}^{n-1} + \alpha^n (\mathrm{d}_y^n - \mathrm{y}^{n-1})$.

Step 5: Test convergence. If $\| \mathrm{x}^n - \mathrm{x}^{n-1} \| < \kappa$ and $\| \mathrm{y}^n - \mathrm{y}^{n-1} \| < \kappa$, stop; Otherwise, let n: =n+1 and go to step 1.

3. Example

The sample network in Fig. 1 is used to demonstrate the validity of the CNDP model under the traffic network with guidance system. The network consists of 16 nodes and 17 links. The link travel time function is BPR: $t_a(x_a) = t_a^0[1+0.15(\frac{x_a}{c_a})]$, and link free time t_a^0 and link capacity c_a are shown in Table 1. The normal length of path $r, \forall r \in R_w, \forall w \in W$ in this study is the travel time between the OD pair w in the user equilibrium. In this tested network, $\alpha = 1$, $\varphi = 1.05\, or\, 1.07$. It is assumed here that there is only one OD pair from origin 1 to destination 12. The expanding cost function of link a is given as below:

$$G_a(x_a) = 2.5(y_a)^2 \tag{16}$$

According to the assumptions in this study, travelers choose the path according to the system optimal rulers under user constrains complying with the guidance. So the total travel time of our network design scenario should be less than that of system optimal or General CNDP under the same conditions. This can

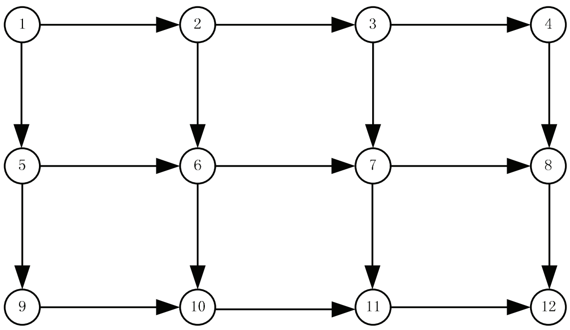

Fig. 1. The tested network

Table 1. The tested network input data

Link	Free Time	Capacity	Link	Free Time	Capacity
(1,2)	20.0	1000	(7,8)	13.0	1000
(2,3)	23.0	500	(5,9)	24.0	800
(3,4)	17.0	500	(6,10)	20.0	500
(1,5)	18.0	1500	(7,11)	26.0	500
(2,6)	19.0	500	(8,12)	19.0	1000
(3,7)	16.0	500	(9,10)	7.0	800
(4,8)	22.0	500	(10,11)	18.0	800
(5,6)	14.0	1000	(11,12)	17.0	800
(6,7)	17.0	1000			

Table 2. The tested network input data

tolerance factor 1.05	tolerance factor 1.07	System-optimal	General-CNDP
186153.44	185934.77	190730.36	190224.84

be easily verified from Table 2. In Table 2, the system travel time is shown for $\varphi = 1.05$, $\varphi = 1.07$. The total travel time for system optimal and the General CNDP under the same budget are also given in Table 2. The enhancement of link after our de network design scenario is shown in Table 3 for $\varphi = 1.05$, $\varphi = 1.07$.

Following, by using of the ratio of the traveler experienced travel time under the enhancement of link capacity to that of user equilibrium before network design, it is analyzed whether the traveler accept the network design scenario. The ratio is called UE unfairness as same as that in Jahn et al. (2005). In Fig. 3, after the enhancement of link capacity, all travelers' travel time is less than that of user equilibrium before network design for $\varphi = 1.05$. When $\varphi = 1.07$, the UE unfairness of traveler is less than 1.04 and 99.7% travelers have better travels time than that of user equilibrium.

From the result of simulation, it can be inferred that if the tolerance factor φ is properly chosen in our CNDP model, travelers all can decrease their travel time after network design than that they experienced at user equilibrium before network enhancement. That is to say, every traveler can profit from our CNDP scenario for proper tolerance factor φ.

Table 3. The enhancement of link after CNDP for different tolerance factor φ

variable	tolerance factor 1.05	tolerance factor 1.07	variable	tolerance factor 1.05	tolerance factor 1.07
$y_{(1,2)}$	10.19	12.23	$y_{(7,8)}$	85.40	71.11
$y_{(2,3)}$	10.74	17.30	$y_{(5,9)}$	29.25	29.81
$y_{(3,4)}$	0	0	$y_{(6,10)}$	3.74	1.63
$y_{(1,5)}$	52.33	56.05	$y_{(7,11)}$	0	0.01
$y_{(2,6)}$	9.57	8.13	$y_{(8,12)}$	111.06	115.05
$y_{(3,7)}$	7.70	4.68	$y_{(9,10)}$	9.26	9.82
$y_{(4,8)}$	0	0.01	$y_{(10,11)}$	88.77	76.47
$y_{(5,6)}$	17.13	19.25	$y_{(11,12)}$	85.43	97.25
$y_{(6,7)}$	28.78	36.95			

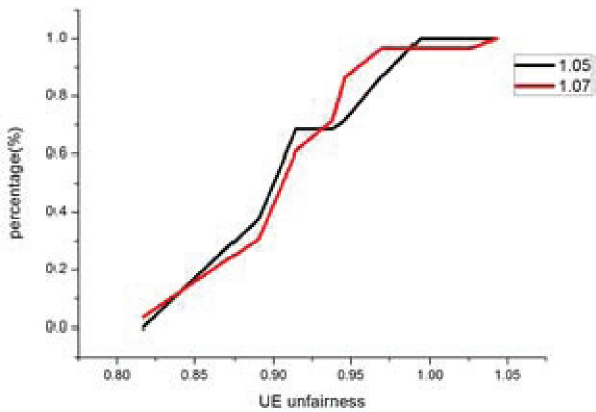

Fig. 2. The distribution of UE unfairness for φ=1.05, φ=1.07

4. Conclusions

This study is based on the research of Jahn et al. (2005) to develop a single level program for CNDP under the traffic network with guidance system. From the result of the test in the example network, if traveler complies with the recommendations provided by guidance devices, then they all profit from our CNDP scenario for proper tolerance factor φ. For further research, we will study how to get proper value of the tolerance factor φ. to make every traveler benefit from our CNDP scenario.

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